

# Long-run Incremental Cost Pricing Based on Nodal Voltage Spare Capacity

Furong Li and E. Matlotse

**Abstract**— *This paper proposes long-run incremental cost (LRIC) pricing to reflect the investment cost in network to maintain the quality of supply, i.e. ensuring that nodal voltages are within limits. The proposed approach makes use of spare nodal voltage capacity or headroom of an existing network (distribution and transmission systems) to provide the time to invest in reactive power compensation devices. A nodal reactive power withdrawal or injection will impact on system voltages, which in turn defer or accelerate the future network investment, the LRIC-voltage network charge aims to reflect the impact on network voltage profiles as the result of nodal reactive power perturbation. This approach provides forward-looking signals that reflect both the voltage profiles of an existing network and the indicative future cost of VAR compensation assets. The forward-looking LRIC-voltage charges can be used to influence the location of future generation/demand for bettering network quality.*

**Index Terms**—LRIC-voltage charges, Long-run incremental cost pricing and RPP problem.

## I. INTRODUCTION

The deregulation of the electric power systems has generated new effort to tackle “green house” effects and binding statutory requirements in encouraging efficient utilization of network assets. Maintaining the quality of supply is one of the key goals required from the network operators. This can be achieved by the use of the reactive power compensation devices in supporting the nodal voltages whilst transporting real power thus improving the efficiency of the network.

The investment costs of maintaining network voltages within statutory limits should be recovered from generators, large industrial customers and suppliers. An approach for network charging to reflect potential impact on network voltages needs to satisfy to purposes:

- 1) to recover capital, operation and maintenance costs of the network VAR compensation assets thus enabling the concerned network establishments to gain a reasonable rate of return on the capital invested;
- 2) to provide forward-looking, economically efficient signals that reflect both the extent of the network VAR

compensation assets required to service withdrawal and/or injection and at the degree of network VAR compensation asset use. This aims to influence the future use of the system by network users to benefit the system voltage profiles.

The majority of the network charging methodologies reflect the investment cost incurred in circuits and transformers to support real and reactive power flow. To reflect investment cost incurred for maintaining network voltages in network charges has received very little attention in network charges [5-9]. Paper [1] presents the first integrated framework for VAR planning and spot-pricing based on marginal costs but it does not have the ability to influence the future use of the system to benefit the network. Other approaches in reactive power pricing tend to reflect the operational cost from new customers, i.e. how they might change network losses [2, 3, 4]. They do not reflect the capital investment cost.

The usage-based MW-Miles or MVA-Miles charging methodologies [7] reflect the extent of the use of the network by network users. The usage-based methodologies were found inefficient as they cannot discriminate between network users who incur additional operating costs or network reinforcement and expansion, and those who reduce otherwise needed network upgrades. It was against this background that the concept of incremental charging methodologies was preferred [5, 6]. The approaches of [8, 9] were employed and they successfully addressed the concerns found lacking in the usage-based methodologies.

This paper proposes an LRIC approach to price the capital cost incurred in the network to support nodal voltages. The approach employs the unused nodal voltage capacity or headroom within an existing network to provide an economically efficient forward-looking pricing signal to direct the siting of future demand and generation. A nodal injection/withdrawal of reactive power will impact on the nodal voltage, the impact will be further propagated over the entire network. The impact on the nodal voltage will affect the investment horizon of network compensation devices. As the LRIC aims to give indicative future investment cost in maintaining system voltage profiles, each study node is a candidate for a reactive compensation device. Depending on the headroom of each study node, the investment horizon for each nodal can be inferred. For a nodal reactive perturbation, there will be a related benefit if the system wide investment can be deferred otherwise there will be a cost if it can be advanced. Then the LRIC-voltage charges are the sum of the

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difference in the present value of the future investment with and without the nodal reactive power injection or withdrawal.

## II. MATHEMATICAL FORMULATION OF LONG-RUN INCREMENTAL COST PRICING BASED ON DC POWER FLOW

The voltage charging principle is based upon the premise that for an assumed nodal generation/load growth rate there will be an associated rate of busbar voltage degradation. Given this assumption the time horizon for a busbar to reach its upper /lower voltage limit can be evaluated. Once the limit has been reached there will be a capital cost associated to keep the nodal level within the statutory limits. A nodal demand/generation increment would affect the reinforcement investment horizon. The nodal voltage charge would then be the difference in the present value of the future reinforcement consequent to voltage with and without the nodal power flow.

The proposed charging model can be implemented through the following steps:

### 1) Evaluating the future cost of network to support an existing customer at node N

If a network node b, has lower voltage limit,  $V_L$  and upper voltage limit  $V_H$ , and supports a voltage  $V_b$ , then the number of years for the voltage to grow from  $V_b$  to  $V_L/V_H$  for a given voltage degradation rate  $v$  can be evaluated from (1.a) or (1.b)

$$\text{for demand: } V_L = V_b \times (1-v)^{n_{bL}} \quad (1.a)$$

$$\text{for generation: } V_H = V_b \times (1+v)^{n_{bH}} \quad (1.b)$$

where:  $n_{bL}/n_{bH}$  is the number of years  $V_b$  takes to reach  $V_L/V_H$ .

Reconfiguring equations (1.a) and (1.b) constitute:

$$(1-v)^{n_{bL}} = \frac{V_L}{V_b} \quad (2.a)$$

$$(1+v)^{n_{bH}} = \frac{V_H}{V_b} \quad (2.b)$$

Taking the logarithm of equations (2.a) and (2.b) on both sides gives

$$n_{bL} \times \log(1-v) = \log V_L - \log V_b \quad (3.a)$$

$$n_{bH} \times \log(1+v) = \log V_H - \log V_b \quad (3.b)$$

then the values of  $n_{bL}/n_{bH}$  are

$$n_{bL} = \frac{\log V_L - \log V_b}{\log(1-v)} \quad (4.a)$$

$$n_{bH} = \frac{\log V_H - \log V_b}{\log(1+v)} \quad (4.b)$$

The assumption is that when the node is fully loaded the reinforcement will take effect. This means that investment will be effected in  $n_{bL}/n_{bH}$  years when the node utilization reaches  $V_b$ . At this point an installation of a VAR compensation asset is regarded as the future investment that will be needed at the node to support the voltage.

### 2) Determining the present value of future investment cost

The function of how far into the future investment can be effected can be determined by evaluating the future investment by discounted it back to its present value. For a set discount rate of  $d$ , then the present value of the future investment in  $n_b$  years will be:

$$PV_{bL} = \frac{Asset_{CbL}}{(1+d)^{n_{bL}}} \quad (5.a)$$

$$PV_{bH} = \frac{Asset_{CbH}}{(1+d)^{n_{bH}}} \quad (5.b)$$

where  $Asset_{CbL}/Asset_{CbH}$  is the modern equivalent asset cost to cater for supporting voltage due to lower voltage limit / upper voltage limit.

### 3) Deriving the incremental cost of an additional power injection or withdrawal at node N

If the nodal voltage change is  $\Delta V_{bL}$  consequent upon an additional withdrawal / injection at node N of  $\Delta Q_m$ , will put forward / delay the future investment from year  $n_{bL}$  to  $n_{bnewL}$

$$\text{for withdrawal } V_L = (V_b - \Delta V_{bL}) \times (1-v)^{n_{bnewL}} \quad (6.a)$$

$$\text{for injection } V_L = (V_b + \Delta V_{bL}) \times (1-v)^{n_{bnewL}} \quad (6.b)$$

$$\text{for withdrawal } V_H = (V_b - \Delta V_{bH}) \times (1+v)^{n_{bnewH}} \quad (6.c)$$

$$\text{for injection } V_H = (V_b + \Delta V_{bH}) \times (1+v)^{n_{bnewH}} \quad (6.c)$$

Equations (7.a), (7.b), (7.c) and (7.d) give the new investment horizon as

$$n_{bnewL} = \frac{\log V_L - \log(V_b - \Delta V_{bL})}{\log(1-v)} \quad (7.a)$$

$$n_{bnewL} = \frac{\log V_L - \log(V_b + \Delta V_{bL})}{\log(1-v)} \quad (7.b)$$

$$n_{bnewH} = \frac{\log V_H - \log(V_b - \Delta V_{bH})}{\log(1+v)} \quad (7.c)$$

$$n_{bnewH} = \frac{\log V_H - \log(V_b + \Delta V_{bH})}{\log(1+v)} \quad (7.d)$$

then the present values of the future investments are

$$PV_{bnewL} = \frac{Asset_{CbL}}{(1+d)^{nbnewL}} \quad (8.a)$$

$$PV_{bnewH} = \frac{Asset_{CbH}}{(1+d)^{nbnewH}} \quad (8.b)$$

The changes in the present values as consequent of the nodal withdrawal / injection  $\Delta Q_{In}$  are given by (9.a) and (9.b)

$$\Delta PV_{bL} = PV_{bnewL} - PV_{bL} = Asset_{CbL} \left( \frac{1}{(1+d)^{nbnewL}} - \frac{1}{(1+d)^{nbL}} \right) \quad (9.a)$$

$$\Delta PV_{bH} = PV_{bnewH} - PV_{bH} = Asset_{CbH} \left( \frac{1}{(1+d)^{nbnewH}} - \frac{1}{(1+d)^{nbH}} \right) \quad (9.b)$$

The annualized incremental cost of the network items associated with component  $b$  is the difference in the present values of the future investment due to  $\Delta P_{In}$  at node N multiplied by an annuity factor

$$IV_{bL} = \Delta PV_{bL} * annuityfactor \quad (10.a)$$

$$IV_{bH} = \Delta PV_{bH} * annuityfactor \quad (10.b)$$

#### 4) Evaluating the long-run incremental cost

The long-run incremental costs to support Node N will be the aggregation of the incremental costs all over the supporting nodes and are

$$LRIC_{NL} = \frac{\sum_{bL} IV_{bL}}{\Delta P_{In}} \quad (11.a)$$

$$LRIC_{NH} = \frac{\sum_{bH} IV_{bH}}{\Delta P_{In}} \quad (11.b)$$

### III. IMPLEMENTATION

The 4-bus network shown in Figure 1, has demand D1 supported by one circuit, while D2 is supported by two circuits. All the circuits are assumed to be identical and the potential VAR compensation assets (SVC) at all the buses are also identical assumed to have the same investment cost of £1,450,000. The circuit maximum rating is 45 MVar and the nodes are at 275kV. The voltage limits are assumed to be  $1 \pm 6\%$  pu. The use of an optimal power flow (OPF) was employed to capture the nodal voltages and the circuit utilization while varying the loading condition of the system.

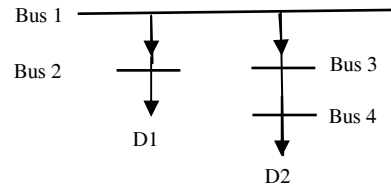


Figure 1. 4-bus network

D1 and D2 were varied from 0 to 45 MVar, constituting a change of 0% to 100%, while monitoring the respective bus voltages and percentage circuit utilization to support buses 2 and 4. The proposed charging approach provides the LRIC voltage charges for demand connecting at bus 2 as shown by fig. 2 and table 1. The charges at bus 2 monotonically increase with increasing circuit utilization. As can be seen from table 1, the voltages are within specified limits. Further, fig.3 shows the charges at buses 2 and 4, while table 3 shows the charges, the circuit utilization, the voltages at buses 2, 3 and 4. In figure 3, buses 2 and 3 have monotonically increasing charges with increasing circuit utilization. Buses 2 and 3 charges are within for the whole ranges of the respective circuit utilizations and voltage limits. On the other hand, Bus 4 charges increases steadily and towards the lower voltage limit they rise sharply. From table 2 it can be seen that at 100% circuit utilization, the lower voltage limit is exceeded and the charges even rise more sharply. This is because reinforcement was supposed to have been effected when the lower voltage limit was reached.

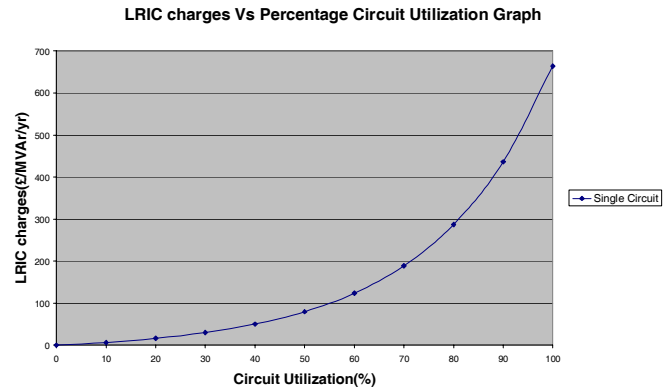


Figure 2. LRIC at bus 2 due to withdrawal.

TABLE 1: LRIC, CIRCUIT UTILIZATION AND VOLTAGE AT BUS 2

% Circuit Utilization	V2	V2 charges (£/MVA/yr)
0	1.06	0
10	1.055	6.68
20	1.05	16.26
30	1.044	30.13
40	1.039	50.27
50	1.033	79.86
60	1.028	123.61
70	1.022	188.78
80	1.017	287.07
90	1.011	436.08
100	1.005	663.99

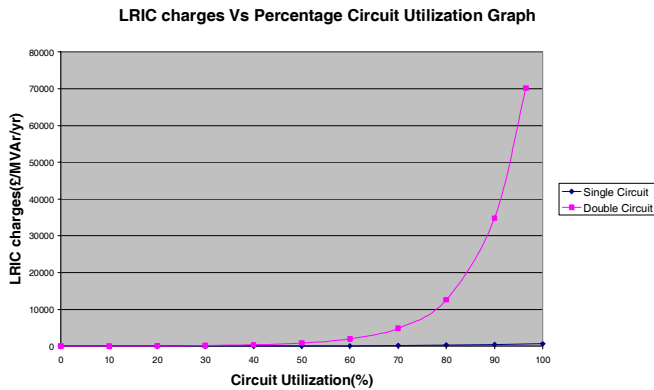


Figure 3. LRIC at bus 2 and bus 4

TABLE 2: LRIC, CIRCUIT UTILIZATION AND VOLTAGE AT BUSES 2, 3 &amp; 4

% Cct Util	V2 pu	V3 pu	V4 pu	V2 chrgs £/MVA/yr	V2 chrgs £/MVA/yr	V2 chrgs £/MVA/yr
0	1.06	1.06	1.06	0	0	0
10	1.055	1.055	1.03	6.68	6.81	25.71
20	1.05	1.049	1.039	16.26	17	75.22
30	1.044	1.044	1.028	30.13	32.57	175.48
40	1.039	1.038	1.016	50.27	56.86	389.35
50	1.033	1.031	1.004	79.86	95.66	870.5
60	1.028	1.025	0.991	123.61	159.22	2015.02
70	1.022	1.018	0.978	188.78	266.18	4899.25
80	1.017	1.01	0.964	287.07	451.79	12635.81
90	1.011	1.002	0.95	436.08	784.87	34829.28
100	1.005	0.994	0.935	663.99	1405.57	420.16

Flipping the single circuit to the left provides figure 4, where a 0% circuit utilization of the double circuit corresponds to 100% of the single circuit. Figure 4 show that the cost to the network will be the same for a node that is supported by double circuits loaded at 31%, or by a single circuit loaded at 69%.

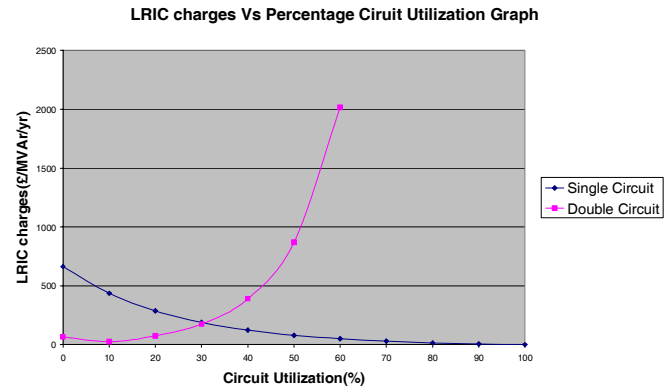


Figure 4: The equilibrium between the nodal extent of use and the degree of circuit utilization.

#### IV. CONCLUSIONS

This paper presents an approach that prices the capital investment cost incurred in the network for keeping the system voltages within statutory limits. The proposed approach is based on a long-run incremental cost (LRIC) pricing developed in Bath, pricing the use of network circuits and transformers by network customers at each node of the network. The most attraction of the approach is that for the first time a method is proposed to allocate the cost of maintaining the quality of supply based on the spare nodal voltage capacity of an existing network. The paper demonstrated a very simple approach to determine the indicative cost of LRIC-voltage charges. The resulting network voltage charging model is reflecting the extent of the network buses utilized by a connected party and the degree of the network nodal utilization. This aims to influence future network users to change their utilization patterns to better network voltage profiles.

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## VI. BIOGRAPHIES

**Dr. Furong Li** (M'2000) was born in Shannxi, China. She received her B.Eng. in Electrical Engineering from Hohai University, China in 1990, and her Ph.D. in 1997 with a thesis on Applications of Genetic Algorithms in Optimal Operation of Electrical Power Systems. She is a senior lecturer in the Power and Energy Systems Group at the University of Bath. Her major research interest is in the area of power system planning, analysis and power system economics.

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