

Storing wind for a rainy day:

What kind of electricity does Denmark export?

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Abstract

On windy days, Denmark tends to export electricity to its neighbours, and to import power on calm days. Storing electricity in this way thus allows the country to deal with the intermittency of wind generation. We show that this kind of behaviour is theoretically optimal when a region with wind and thermal generation can trade with one based on hydro power. However, annual trends in Denmark's trade follow its output of thermal generation, Nordic production of hydro power, and the amount of water available to Scandinavian generators, not wind generation. We estimate the cost of volatility in Denmark's wind output to equal between 4% and 8% of its market value.

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1. Introduction

The last decade has seen a remarkable increase in the number of wind installations throughout the world, as part of a coordinated effort to shift towards a higher share of renewable generation. In Europe, Denmark is amongst the leading countries in wind generation, in terms of installed capacity relative to population and demand (Cossent et al, 2009; Gøransson and Johnsson, 2008; Eriksen et al., 2005). It is therefore not surprising that the country has often been used as a case study to investigate the consequences and challenges of high rates of wind generation, from both a technical and economic perspective.

The Danish electricity system is divided between two separate zones, East and West, which are synchronized to the NORDEL (Nordic) and UCTE (Western European) grid systems, respectively. Although East Denmark is the more densely populated zone of the two, it is West Denmark that exhibits the highest rates of wind generation (21% as opposed to 8% for East Denmark, Østergaard 2008). The two zones are independent from each other, in the sense that they cannot communicate for the purposes of power balancing or wind integration. In 2003, towards the middle of our period, Western Denmark had approximately 2400 MW wind power installed as opposed to Eastern Denmark with a more modest 1100MW, which are expected to rise to 3860 MW and 1850 MW, respectively, by the year 2020 (Lund and Münster, 2003).

The management of the highly intermittent wind-generated output in a country with practically no domestic capacity for hydro storage, like Denmark, is of great interest to the UK and other regions that are committed to increase their use of renewable generation significantly, and likely to obtain much of the increase from wind power. Sharman (2005) argues that the Danish experience is unique and very hard to replicate, as it relies heavily on the availability of “privileged” interconnections to Norway and Sweden, both equipped with large hydro-generation capacities, and therefore able to adjust their output very rapidly to

changing flows of wind output. Moreover, a high correlation coefficient between wind output and net power outflows may suggest that a significant part of the wind-generated output is exported. Bach (2009) calculates that 77% of the power produced in East Denmark in 2006 was exported, while CEPOS (2009) gives an eight-year average for West Denmark of 57%. For CEPOS, the Danish consumers who are subsidising wind power are not seeing the benefit of the low-carbon electricity it produces. An alternative view, presented by Mignard et al (2007), is that this claim is “unwarranted”, because there is a stronger correlation between the production of electricity from (so-called) local Combined Heat and Power plants and exports from West Denmark than there is between wind production and exports.

Our aim is to examine Denmark’s electricity production and trade, on different timescales, in the context of an economic model that shows how wind generation should be co-optimised with hydro and thermal plants. In our setting, the wind and thermal plants are in one country (i.e. Denmark), and the hydro stations in another (i.e. Norway). We will show in the model that the average level of thermal generation and of exports from our wind-thermal country is inversely correlated with the amount of hydro generation available, and that this result is reflected in monthly data for Denmark between 1996 and 2008. We will also show that short-term fluctuations in the level of wind generation are highly correlated with exports from the wind-thermal country, and that this is also reflected in hourly data from 2001 to 2009.

The rest of the paper is organised as follows: Section two summarises the related literature. Section three develops a theoretical model to explain the trade flows between a wind- and hydro generating region. The model extends Førsund’s (2007) generalised bathtub framework, making easier to account for the impact of varying wind output on the trade flows between the two regions of interest. Section four tests the theoretical implications of the model for the volumes of production and trade, using data for Denmark and its neighbours.

The fifth section assesses the cost of fluctuations in wind output, in terms of differences between average prices and those actually received for wind output and Danish exports. Finally, section six concludes.

2. Previous Work

Most of the studies on the integration of wind-generated output with other forms of generation appear in the engineering literature and focus on the technical challenges entailed in the integration and storage of large amounts of highly variable and intermittent wind output. In the economics literature, the majority of the studies on wind generation analyse the possible impact of renewables on prices for a particular country or region. For Denmark, Østergaard (2008) models the impact of the geographic dispersion of wind generation on the variability of wind output and, consequently, the need for operational reserve capacity. His findings suggest an explicit negative correlation between the two, implying that local fluctuations in wind output can be partly offset by spreading wind turbines over a sufficiently large area¹.

The benefits of interconnection in the integration of wind output are also discussed in Oswald et al (2008) for the UK, who, however, find that UK wind speeds can be highly and positively correlated with these in neighbouring countries during times of low output. As a result, wind output can be low during periods of high demand, implying again the need for sufficient available reserve capacity.

Rosen et al (2007) present a model analysis of the short- and long-term effects of high levels of wind generation on output composition for Germany. Their results suggest that wind output acts mainly as a substitute for intermediate- and base-load plants (such as coal, lignite and nuclear plants), whereas gas-fired plants are used as stand-by capacities to balance fluctuations in wind output. Similar findings are reached in Weigt and Hirschhausen (2009),

¹ However, the argument cannot be readily generalised to reduce the maximum reserve capacity needed in the area, as Østergaard acknowledges the presence of times with zero wind production in all interconnected areas.

who use hourly wind generation data for Germany over the period 2006 to 2008, to assess the potential of wind energy as a substitute of installed conventional generation capacities. They also discuss the impact of wind generation on market prices, finding it to lead to significant reductions, especially during peak hours.

3. The Model

The model we use is an extended version of the multi-period “bathtub” framework presented in Førsund (2007) and Førsund et al (2008), adjusted to allow for the inclusion of thermal, wind and hydro generation over several time periods, indexed by $t = 1 \dots T$. In its simplest form, the original model provides a stylised way to analyse the effect of competing hydro and thermal generators on prices and demand patterns for each type of plant². The quantities of generating output for each type of plant (and, the resulting price of electricity) can be obtained as solutions to a constrained optimisation problem that maximises a social welfare function subject to a series of capacity constraints.

We assume that generation takes place within two regions, A and B. The two regions are connected to each other with an interconnector which, for simplicity, we assume has unlimited capacity (so that trade will equalise their prices). Region B is equipped with hydro plants of total fixed capacity \bar{g}^h . Region A produces thermal and wind-generated output, with total fixed capacities of \bar{g}^{Th} and \bar{g}^w , respectively. The thermal generators have a collective variable cost function given by $\phi_t(g_t^{Th})$. Hydro and wind generators have no variable costs. While the wind capacity is fixed, its ability to generate electricity depends on the strength of the wind, represented by a stochastic variable $\omega_t \in [0,1]$. All generators behave competitively.

² In a more recent article, Førsund et al (2008) extend the original model to include wind and hydro generation (but not thermal).

There is demand in each region, D_t^A p_t^A and D_t^B p_t^B , which varies between periods and is sensitive to the wholesale price in that region in that period. We can invert the demand functions to obtain the marginal value of power, given by p_t^A D_t^A and p_t^B D_t^B . Free trade between the two regions will eliminate any arbitrage opportunities and therefore in equilibrium there will be only one price p_t^* across the two regions.

We assume that the thermal and hydro producers act as price takers and choose how to allocate their outputs, g_t^{Th} and g_t^h , between the periods in such a way as to maximize their total profits subject to their capacity constraints. From a social planner's point of view, therefore, the optimisation problem at hand takes the form:

$$\max_{g_t^{Th}, g_t^h, g_t^w, D_t^A, D_t^B} \sum_{t=1}^T \int_0^{D_t^A} p_t^A q dq + \int_0^{D_t^B} p_t^B q dq - \phi_t g_t^{Th} \quad (1)$$

subject to

$$g_t = g_t^h + g_t^{Th} + g_t^w \geq D_t^A + D_t^B \quad (1.1)$$

$$g_t^h \leq \bar{g}^h \quad (1.2)$$

$$g_t^{Th} \leq \bar{g}^{Th} \quad (1.3)$$

$$g_t^w \leq \omega_t \bar{g}^w \quad (1.4)$$

$$\sum_{t=1}^T g_t^h \leq W \quad (1.5)$$

The first two integrals in the objective function represent the value of electricity consumption in the two regions, and the variable cost of thermal output is subtracted from this.

The energy balance constraint (1.1) is written as an inequality, but in equilibrium, as in Førsund (2007), generators will not spill output. Constraints (1.2)-(1.4) reflect binding capacity constraints faced by each producer – the per-period output from hydro and thermal stations must be less than their capacity, while the output from the wind generator must be

less than its capacity multiplied by the strength of the wind in that period, ω_t . The last equation (1.5) is the water balance constraint facing the hydro producer. It must allocate a fixed amount of water, W , across the periods. We assume that the amount of water available is known at the start of the optimisation, and that there are no other reservoir constraints.³ If reservoir and transmission constraints do not bind, then storing water can eliminate arbitrage opportunities and impose a common price across regions and time periods. The objective Lagrange function for maximisation takes then the form:

$$\begin{aligned} \max_{g_t^{Th}, g_t^h, g_t^w, D_t^A, D_t^B} L = & \sum_{t=1}^T \int_0^{D_t^A} p_t^A q dq + \int_0^{D_t^B} p_t^B q dq - \phi_t g_t^{Th} \\ & - \lambda_t (D_t^A + D_t^B - g_t^h - g_t^{Th} - g_t^w - \theta_t^{Th} \bar{g}^{Th} - g_t^{Th} + \theta_t^h \bar{g}^h - g_t^h + \theta_t^w \omega_t \bar{g}^w - g_t^w \\ & + \mu \left(W - \sum_{t=1}^T g_t^h \right) \end{aligned} \quad (2)$$

The top line is the welfare function from equation (1), the second line gives the per-period constraints (energy balance, and generation from each type of plant below its effective capacity), and the third line gives the inter-temporal constraint on hydro generation. The first order conditions include:

$$\frac{\partial L}{\partial D_t^A} = p_t^A(D_t^A) - \lambda_t \leq 0 \quad (2.1)$$

$$\frac{\partial L}{\partial D_t^B} = p_t^B(D_t^B) - \lambda_t \leq 0 \quad (2.2)$$

$$\frac{\partial L}{\partial g_t^{Th}} = \lambda_t - \phi_t' g_t^{Th} - \theta_t^{Th} \leq 0 \quad (2.3)$$

$$\frac{\partial L}{\partial g_t^h} = \lambda_t - \theta_t^h - \mu \leq 0 \quad (2.4)$$

$$\frac{\partial L}{\partial \mu} = W - \sum_{t=1}^T g_t^w \geq 0 \quad (2.5)$$

³ For simplicity we assume that reservoir constraints are not binding. A more generalised discussion of the impact of reservoir constraints on the optimization set up can be found in Førsund (2007).

$$\frac{\partial L}{\partial \theta_t^w} = \omega_t \bar{g}^w - g_t^w \leq 0 \quad (2.6)$$

Equations 2.1 and 2.2 ensure that, in the absence of transmission constraints, the prices in the two regions will be equal within each period. The prices must equal system lambda, the marginal cost of providing power. The next two equations show how that marginal cost relates to the opportunity costs of different kinds of capacity. Equation 2.3 requires system lambda to equal the opportunity cost of thermal plant in that period, given by the derivative of its variable cost plus the shadow value of the capacity constraint on thermal plant in that period – if the latter is binding.

Equation 2.4 comes from the hydro plant, which has an opportunity cost given by the shadow value of its capacity constraint in that period and the shadow value of water – which does not vary across periods. In the absence of binding capacity constraints, this means that system lambda will be the same in each period, equal to the shadow value of water, and that this will apply in both regions.⁴ This then means that the marginal cost of thermal generation should be the same in all periods. If the cost function is invariant over time, this means that the output from thermal plant should also be the same in both high- and low-demand periods. The thermal region (A) would thus tend to import power in high-demand periods, and export it when demand is low. If the thermal cost function does vary over time, then there will be more thermal output in periods when generation costs are low.

⁴ It would be straightforward to add a constraint, and shadow value, for the capacity of the interconnector between the regions. The two regional prices would diverge whenever the flow needed to equalise them exceeded the interconnector capacity. We chose to keep the model as simple as possible, while still covering the key issues relevant to this paper.

Equation 2.5 reproduces the water balance constraint, which will hold with equality. Similarly, equation 2.6 will hold with equality, ensuring that the potential wind output is all used in each period, unless it exceeded the level of demand at a price of zero!

The shadow value of water is the most important variable in determining the level of output. In the absence of relevant binding constraints, this value will equal system lambda, and hence the marginal cost of thermal generation and the marginal value of demand, in each period. If more water was available, its shadow value would fall, and the lower system lambda would show both that demand would increase and that the marginal cost of thermal output would decrease, because less was required, crowded out by the hydro generation.

$$\sum_{t=1}^T D_t^A \mu + D_t^B \mu - \omega_t \bar{g}_t^w - g_t^{Th} \mu - W = 0 \quad (3)$$

Equation (3) adds up all the energy balance constraints and substitutes for wind and hydro generation, assuming no other constraints are binding. If so, we can differentiate it totally to obtain:

$$\frac{\partial \mu}{\partial W} = \frac{-1}{\sum_{t=1}^T \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu} \right)} \quad (4)$$

where the first term in the denominator is the inverse of the slope of the marginal cost function for thermal generation, thus giving the increase in output obtained for an incremental rise in price. A change in wind output (in any period) will also affect the shadow value of water, with an almost identical formula:

$$\frac{\partial \mu}{\partial \omega_t \bar{g}^w} = \frac{-1}{\sum_{t=1}^T \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu} \right)} \quad (5)$$

We can calculate the net exports from our thermal-wind country (country A), adding up the relevant items:

$$X_t^A = \omega_t \bar{g}_t^w + g_t^{Th} \mu - D_t^A \mu \quad (6)$$

If the amount of water available in the hydro system (country B) changes, this will lead (in the absence of other binding constraints) to an equal change in country A's net exports in each period:

$$\frac{\partial X_t^A}{\partial W} = \frac{- \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} \right)}{\sum_{t=1}^T \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu} \right)} \quad (7)$$

In the simplified case in which country B's demand for power is completely insensitive to price and the sensitivity of the thermal generators' marginal cost to output does not change over time, a one unit increase in the amount of water available in country B will reduce country A's exports by $1/T$ units in each period. The most efficient way of using the water is to distribute it evenly over the periods, and if none of it is to be used inside country B (demand is not dependent on the price), then all can be used to allow a higher demand or a lower level of thermal generation in the other country. If the amount of wind available in country A changes in period t , then this will affect its net exports differently in that period and in any other period, denoted s :

$$\frac{\partial X_t^A}{\partial \omega_t \bar{g}^w} = 1 - \frac{\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu}}{\sum_{t=1}^T \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu} \right)} \quad (8)$$

$$\frac{\partial X_s^A}{\partial \omega_t \bar{g}^w} = \frac{- \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} \right)}{\sum_{t=1}^T \left(\frac{1}{\phi_t'' g_t^{Th}} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu} \right)} \quad (9)$$

On the same simplifying assumption as before (demand in country B is completely price-insensitive), a one unit increase in the amount of wind available in country A will lead to an increase in its exports of $1 - 1/T$ in that period, and a reduction of $1/T$ in every other period. In other words, most of the additional wind output is exported, substituting for hydro production in country B, but this then increases the amount of water available for use in other periods, reducing its shadow value and allowing hydro generation to substitute for thermal generation in country A. With a fixed amount of water and a fixed demand for power in country B, all of the exported power would eventually be re-imported. We illustrate some of these results graphically in the appendix.

Note that if the amount of wind generation capacity increases, as opposed to the amount of output within any one period, this will lead to an expected reduction in the output from the thermal plants in every period, and hence a reduction in the shadow price of water.

It can therefore be seen that, in such a setting, “exporting” (in the sense of “storing”) wind output to a reservoir to “import it back” when demand is peaking, becomes a defensible and, in fact, a socially optimal strategy. The next section looks into the trade patterns between Denmark and Norway and investigates to what extent the flows of output between the two countries could be justified in the context of the model that has been outlined in this section.

4. Patterns of production and trade

In this section we assess the practical implications of our model, using data to analyse the trade flows of thermal-, hydro- and wind-generated output between Denmark and its neighbours. Our dataset compiles information from the Danish Transmission System Operator, Energinet (www.energinet.dk), for hourly observations on the production, consumption and trade for East and West Denmark, from 2001 to 2009; and from Nordel (www.entso.eu), for monthly data on Nordic reservoir levels, output and trade from 1996 to 2008. Both sources give Danish production from three types of generating plant: wind, local CHP plants (which tended to follow their heat demand rather than participating actively in the electricity market) and primary plants, also usually CHP, but larger, and active participants in the wholesale market. Trade flows are given separately for each of the five interconnectors: Norway to West Denmark, and Sweden and Germany to (both) East and West Denmark.

Figure 1 goes about here

We start with the big picture, trends in production and exports on an annual scale. Our hypothesis is that thermal production will tend to be inversely correlated with the amount of water available to hydro generators. Figure 1 gives twelve-month moving totals for thermal and wind production in Denmark (left hand scale) and hydro inflows and production in Norway, Sweden and Finland (right hand scale), derived from Nordel's annual statistical yearbooks.⁵ At the top of the figure, the hydro inflows are clearly volatile, even after taking 12-month totals. The output levels are smoother, but also closely follow the inflows, with a bit of a lag (the industry cannot take advantage of a wet year until after it has occurred).

⁵ The Nordel yearbooks give the reservoir levels in each country at regular intervals through the year, together with monthly hydro production, from which it is straightforward to derive the inflows.

In the middle of the diagram, Denmark's production of thermal electricity follows the inverse pattern to Nordic hydro generation – when the latter is high, Denmark generates relatively less power. Note that the scales of the two series are different, and an equal distance on the page implies two and a half times as much hydro generation as thermal: the change in Denmark's generation is only part of the response to hydro conditions further north.

At the bottom of the diagram, the 12-month average of Denmark's net exports clearly follows the same pattern as the country's thermal generation. When thermal generation is high, so are exports. There is no apparent correlation between the level of exports and of wind generation, however. Wind output has been gradually increasing throughout the period. Exports and thermal output have cycled around a trend which appears essentially flat since 2000.

These visual impressions are confirmed by analysis of the correlations between the variables, shown in table 1. The top panel gives correlation coefficients between annual values for the whole sample (1996 to 2008), while the bottom panel drops the first four years, in which wind generation was very low (the industry was in its infancy) but exports were (for this period) unusually high.

Table 1: Correlation coefficients between Danish power and Nordic hydro variables

1996-2008	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.824	-0.825	-0.781	0.351
Hydro output		1	-0.887	-0.815	0.337
Danish exports			1	0.980	-0.580
Danish thermal				1	-0.726
Danish wind					1
2000-2008	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.783	-0.776	-0.788	0.144
Hydro output		1	-0.902	-0.831	-0.096
Danish exports			1	0.976	-0.045
Danish thermal				1	-0.242
Danish wind					1

Source: authors’ calculations from Nordel data, using calendar year totals

It is clear that water inflows and hydro generation are highly correlated, and that there is a strong negative correlation between these variables (especially the latter) and Denmark’s exports. There is a very high correlation between Denmark’s thermal generation and its exports, and basically no correlation between wind generation and any of the other variables in the period after 2000.

Moving to an intermediate scale, figure 2 shows the monthly pattern of production and trade from January 2001 to December 2009. Denmark clearly produces more electricity from both wind and thermal stations in the winter months, when demand is also higher. The country also exports more power in the winter, on average. This is because most power stations in Denmark also provide district heating during the winter, effectively reducing their marginal cost of producing electricity. This movement in their cost function is big enough for

the seasonal change in their (presumably optimised) output to be greater than the change in demand and so Denmark is usually a net exporter in the winter months, importing power in the summer.

Figure 2 to go about here

Moving to the most detailed scale, figures 3a and 3b show the average hourly pattern of Denmark's output and consumption over the day in the months of June to September (summer) and November to February (winter) – demand is clearly higher in the daytime than overnight. In the summer, but not the winter months, the output from wind plants is higher during the day than the night, (10 pm to 6 am), as is the output from the large primary stations. The stations with the greatest variation in production are the smaller local stations, which tend to follow their heat loads rather than market signals – their daytime output was 60% above their night-time production. Total production exceeds demand, on average, in most hours of the day, as Denmark was a net exporter during this period. The difference is greater when demand is low, again consistent with our model – the thermal producer is most likely to export off-peak.

Figure 3a and 3b to go about here

The relationship between net exports, generation output and consumption is also summarised in the three panels of table 2. Panel 2.1 presents the correlation coefficients between our main variables, using monthly averages drawn from the full sample. Net exports are highly correlated with the output of thermal plants, with primary stations having the highest (nearly perfect) correlation coefficient. The correlation between consumption and thermal output is

also strong throughout the sampled period. The strong correlation between heat and electricity demand means that local CHP plants have a particularly high correlation. Panels 2.2 and 2.3 present the correlation coefficients for the same variables, but now taken across the 24 hours of the day, in two seasons. Thus, panel 2.2 presents the underlying daily pattern for winter months (November to February), whereas panel 2.3 focuses on summer months (June to September). The winter season shows strong negative correlation between net exports and both forms of thermal output, as well as consumption – a pattern that is consistent with off-peak exporting. Consumption in this season shows strong positive correlation with all three forms of generation output, though it is clearly strongest for thermal plants. The correlation coefficients for the summer season, on the other hand, provide a more mixed picture. The correlation between net exports and thermal output and consumption is relatively weak, though the sign of the coefficients still suggests a “weaker” off-peak pattern of exporting behaviour.

The correlation between the average daily pattern of wind output and net exports is negative in both seasons, (although very weak in the summer) – the wind tends to be stronger in the daytime, the peak hours when Denmark tends to import more power (summer) or export less (winter).

Table 2: Correlations between average patterns

2.1 Monthly averages - full sample, hourly averages

	Net exports	Primary	Local	Wind	Consumption
Net exports	1	0.941	0.785	0.415	0.736
Primary		1	0.686	0.185	0.701
Local			1	0.361	0.867
Wind				1	0.555
Consumption					1

2.2 Hourly averages - winter months

	Net exports	Primary	Local	Wind	Consumption
Net exports	1	-0.944	-0.943	-0.479	-0.973
Primary		1	0.973	0.614	0.991
Local			1	0.637	0.988
Wind				1	0.606
Consumption					1

2.3 Hourly averages - summer months

	Net exports	Primary	Local	Wind	Consumption
Net exports	1	-0.403	0.117	-0.127	-0.331
Primary		1	0.797	0.697	0.985
Local			1	0.553	0.856
Wind				1	0.756
Consumption					1

The analysis so far has been in terms of averages, but how does the volatility of wind generation affect Denmark? We represent this volatility by the deviation of hourly wind output (and other variables) from the average level for that hour in that month. Indexing hours by h , days by d and months by m , and with D_m days in a month, we calculate these average levels by $\bar{q}_{mh}^{-i} = \sum_{d=1}^{D_m} \left(\frac{1}{D_m} \sum_{d=1}^{D_m} q_{mdh}^i \right)$, where the output in hour h of day d of month m from plants of type i is given by q_{mdh}^i . The deviations, \tilde{q}_{mdh}^i , are simply given by $\tilde{q}_{mdh}^i = q_{mdh}^i - \bar{q}_{mh}^{-i}$.

Table 3 shows that deviations from the normal level of net exports are strongly and positively correlated with deviations in wind output from its mean value. This in turn suggests that Danish exports are on average higher when there is “excess wind”. Although the correlation of net exports with thermal output is positive, it is significantly weaker. Finally, deviations in Danish exports are found to be uncorrelated with deviations in consumption or

the output from local CHP plants. Yet again, this is in line with the model of the previous section.

Table 3: Correlation coefficients between deviations from normal levels

	Net Exports	Primary Stations	Local CHP	Wind	Consumption
Net Exports	1	0.280	0.046	0.673	0.079
Primary Stations		1	0.149	-0.342	0.636
Local CHP			1	-0.038	0.192
Wind				1	0.033
Consumption					1

There is a strong correlation between deviations in consumption and in the production from primary stations, suggesting that variations in demand are mostly met by changing the scheduled output from large power stations, with much weaker responses from local stations or exports. The deviations in output of primary stations are negatively correlated with those of the wind stations, implying that the primary stations also respond to high levels of wind production, making room for this in the market. In our model, short-term variations in wind output and in demand should all be dealt with mainly by changing the level of hydro output (and hence net exports), but if the transmission system is constrained, then the output of thermal plants will have to respond instead.

5. The cost of storing wind abroad

The theoretical model and data analysis presented in the earlier sections make clear that there is (and should be) a negative correlation between the level of thermal output in Denmark and

the amount of water available in Norwegian reservoirs. In the short term, there is also a clear relationship between the amount of wind generation and the net exports of electricity from Denmark. Effectively, Denmark is storing some of its wind in its neighbours' power systems. Does it have to pay for the privilege? When the wind is high, power prices will be relatively low (for the time of day and time of year), whereas they will tend to be relatively high when there is little wind. Twomey and Neuhoff (2010) discuss some of the implications of this within a single electricity market, in that wind generators will tend to receive less than the time-weighted average price of power, adjusted for their seasonal output patterns.⁶ Denmark could suffer from a similar effect, internally and with the wind power that it exports to its neighbours. In this section we use data on the value, volume and prices of power flows between the two Danish markets (East and West) and its neighbouring trade partners (Norway, Sweden and Germany) to recover estimates of this cost of “storing wind”.

First, we compare the average price that would be received per MWh of wind power in Denmark if sold in Nord Pool at the prevailing local price,⁷ to a series based on the same prices, but weighted by the average amount of generation in each hour in each month. Indexing hours by h , days by d and months by m , and with D_m days in a month, this smoothed series (calculated for a year at a time) is given by

$$\bar{p}^s = \sum_{m=1}^{12} \sum_{h=1}^{24} \sum_{d=1}^{D_m} p_{mdh} \left(\frac{1}{D_m} \sum_{d=1}^{D_m} q_{mdh}^w \right) \div \sum_{m=1}^{12} \sum_{h=1}^{24} \sum_{d=1}^{D_m} q_{mdh}^w, \text{ where the price and wind output in hour } h \text{ of}$$

day d of month m are given by p_{mdh} and q_{mdh}^w respectively. This smoothed series captures variations in the average pattern of wind output over the year and over the day, but excludes day-to-day variation around this pattern. For the series giving the actual output-weighted

⁶ Green and Vasilakos (2010) simulate this effect for the case of Great Britain in 2020, when wind generation might make up around a quarter of generation, finding that onshore generators might receive 8% less than the time-weighted average price.

⁷ Note that most Danish wind generators receive a feed-in tariff with pre-set rates, rather than the wholesale market prices we present here.

price, \bar{p}^a , replace the bracketed term by q_{mdh}^w . We calculate these prices for each year from 2001 to 2009, and the results are shown in table 4.⁸

Table 4: Prices for wind generation in Denmark, 2001-2009, DKK/MWh

	West Demark				East Denmark			
Year	Time-weighted	Wind: smoothed	Wind: actual	penalty	Time-weighted	Wind: smoothed	Wind: actual	penalty
2001	176.87	178.22	169.34	5.0%	175.44	177.16	173.91	1.9%
2002	189.22	191.45	171.48	10.6%	212.36	214.21	207.35	3.2%
2003	250.27	252.26	214.07	15.3%	273.45	275.69	261.70	5.1%
2004	214.28	213.52	201.86	5.4%	210.94	210.84	206.92	1.9%
2005	277.44	267.29	245.04	8.0%	251.89	250.71	235.85	5.9%
2006	329.54	320.37	302.43	5.4%	361.97	349.71	334.59	4.2%
2007	241.37	238.33	213.49	10.3%	245.94	243.63	226.95	6.8%
2008	420.70	409.31	381.62	6.6%	422.28	411.55	396.02	3.7%
2009	268.41	272.03	256.49	5.8%	296.90	301.48	295.68	2.0%
All years	263.15	265.49	244.38	8.0%	272.38	278.52	267.42	4.1%

Source: energinet data and authors' calculations

We report results for West and East Denmark separately, since the former has a higher proportion of wind generation. In some years, the smoothed average price for wind generation is slightly above the time-weighted average price of power. This is not surprising,

⁸ We use Danish Kroner, noting that 1Euro was worth 7.45 DKK on average during our sample period, with minimal variation. The highest monthly average exchange rate from January 1996 to December 2009 was less than 1.5% above the lowest.

since wind output is higher in the winter months, when we would expect also to see higher prices. In all years, the price weighted by the actual pattern of output is below the smoothed series. The difference captures the effect of the co-variance between the level of wind output and the wholesale price – when wind generation is above average (for the time of day and year) the price is likely to be below average, as noted by Twomey and Neuhoff (2010). The effect is much greater for West Denmark than for East Denmark, where there is proportionally less wind generation. This financial cost of intermittency in West Denmark varies between 5% and 15.3% of the time-weighted average price, while in East Denmark it varies between 1.9% and 6.8%. Over the eight year period, the average cost in West Denmark is 8% of the time-weighted average price, and that in East Denmark is 4.1%. These costs are noticeable but not unbearable.

Second, we estimate the cost of storing wind power in Denmark's neighbours. We will assume that all of the variation in wind output (relative to the hourly-monthly mean, as before) leads to an equal variation in net exports.⁹ Over the course of a month, these quantity variations sum to zero, by definition. If Denmark received the same price for the positive variations as it paid for the negative ones, then these variations would be costless to the country. If, however, the price is lower at times when output is high, then coping with short-term fluctuations in wind output by trade may be expensive.

We value each trade flow at the average of the price in (the relevant part of) Denmark and that in the foreign market – either Nord Pool (for Norway and Sweden) or EEX (for Germany). We use Bialek's (1996) tracing algorithm to identify the flows that are transits, simply passing through Denmark from one neighbouring country to another. This algorithm simply apportions the power leaving a point on a network in proportion to the power entering it, so that if Denmark is importing 300 MW of power from Germany, but exporting 400 MW

⁹ To the extent that not all of the variation in wind output is exported, this over-estimates the cost of storing wind, but it ignores any impacts from variations in the monthly averages.

to Norway and 400 MW to Sweden, we will divide the flow from Germany equally between the two receiving countries. Denmark itself would thus be exporting 250 MW to each country. The price we apply to value the deviation in wind output for that hour is the 50-50 mix of the prices for trade with Norway and with Sweden. (Overall, in this case, the final value will be 50% of the Danish price, 25% of the Norwegian, and 25% of the Swedish.) We denote this price as p_{mdh}^t . The estimated cost of dealing with intermittency through trade is then given by $c^t = \sum_{m=1}^{12} \sum_{h=1}^{24} \sum_{d=1}^{D_m} p_{mdh}^t \tilde{q}_{mdh}^w = \sum_{m=1}^{12} \sum_{h=1}^{24} \sum_{d=1}^{D_m} p_{mdh}^t q_{mdh}^w - \bar{q}_{mh}^w$ where the quantities are those of the wind power.

Table 5: The cost of variations in exports, 2001-2009

Year	West Demark			East Denmark		
	trade cost, m DKK	value of wind output, m DKK	proportion	trade cost, m DKK	value of wind output, m DKK	proportion
2001	17.9	601	3.0%	2.3	161	1.5%
2002	45.0	725	6.2%	5.8	223	2.6%
2003	117.3	1,092	10.7%	15.2	327	4.7%
2004	22.1	1,045	2.1%	3.5	360	1.0%
2005	19.7	1,393	1.4%	13.7	401	3.4%
2006	51.4	1,521	3.4%	13.2	539	2.4%
2007	79.9	1,342	6.0%	23.5	396	5.9%
2008	88.7	2,184	4.1%	22.1	754	2.9%
2009	38.6	1,375	2.8%	12.3	471	2.6%
Totals	480.6	11,279	4.3%	111.6	3,633	3.1%

Source: energinet data and authors' calculations

Table 5 shows the results, giving the value of c^t in each year from 2001 to 2009, separately for East and West Denmark. It also gives the value of wind output, measured at the time-

weighted average price for each year (as in table 4). This shows that the cost of exporting power at times when prices are relatively low, and re-importing it at higher prices (on average) was between 1.4% and 10.7% of the annual value of the wind output in West Denmark, and between 1.0% and 5.9% of the value of the wind output in East Denmark. The average across the entire data sample in West Denmark was 4.3%, and 3.1% in the east. The high cost for 2003 is an outlier, likely to have been affected by exceptionally high prices in Norway in the early months of that year, following a drought the previous autumn. If we exclude that year, the average cost for West Denmark falls to 3.6% of the value of its wind output, measured at the time-weighted annual average prices.¹⁰ A cost of almost DKK 600 million over ten years is clearly noticeable, but it is, again, small compared to the overall value of the wind output. With 54.9 TWh of wind generation from 2001 to 2009, the cost comes to DKK 10.78/MWh (€1.45/MWh) of wind output.

6. Conclusions

There have been suggestions that much of Denmark's output of wind power is exported, with the possible implication that Danish consumers therefore do not benefit from the low-carbon electricity that it produces. In this paper, we present a theoretical model, adapted from the work of Førsund (2007), in which we show how production by thermal and hydro generators should be optimised to take advantage of wind power. A thermal producer should generate more, relative to its demand, at off-peak times, when the level of water available to the hydro producers is low, and when wind output is low. The last is a transient effect, given the variability of the wind.

Our analysis of the Danish experience shows that this model works on all scales.

First, we have shown that Danish exports of power respond to the hydrological situation in

¹⁰ The cost in the East also falls, but by less, to 2.9%, as the cost of trade fluctuations in 2003 was less exceptional for that half of the country.

Norway, Sweden and Finland, and that exports are greatest when hydro generation is relatively low, because inflows of water have been low. This year-to-year pattern of exports is driven by the amount of power that Denmark's fossil-fuelled thermal power stations produce.

At a daily level, Denmark's net exports of power are higher at off-peak times, showing that the country takes advantage of its neighbours' hydro storage to effectively flatten its load-curve. It exports more overnight, and less during the day-time peak, thereby reducing the fluctuations in its own power output. There is a tendency for wind generation to be higher during the day, when Denmark's net exports are relatively lower.

However, the critics of the Danish experience are correct to say that Denmark relies heavily on its neighbours to absorb short term fluctuations in its wind output. We have shown that the correlation between short-term deviations (from the norm for that time of day and year) in wind power and in net exports are much higher than for thermal generation. When it is windy in Denmark, the country's exports rise. What the critics seem not to appreciate, however, is that much of this power is stored in the countries further north, which reduce their own generation to absorb it, and then bought back at times of relatively low wind output. In other words, Denmark tends to store its wind in the form of water.

This would not be possible without the cooperation of its neighbours, and, indeed, the presence of neighbours with large systems and (to the north) a lot of storage hydro. Furthermore, the Danes do have to pay for this storage, in that the positive deviations in net exports are traded at lower prices, on average, than the negative ones. We estimate the cost of this effect at just under DKK 11/MWh of wind output over the period between 2001 and 2008. This is 4% of the value of that output, based on annual time-weighted average market prices. Denmark is therefore able to deal with the problem of intermittency in a reasonably cost-effective way.

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Appendix: A graphical presentation

We can illustrate some of our results graphically, adapting Forsund's (2007) presentation. In figure A1, start with the panel to the right of the thick vertical axis, representing the off-peak period. The first downwards-sloping line gives the demand in region B, the area with hydro production, $D_o^B p_o^B$, while the second line, further from the vertical axis, gives the total demand in the two regions together, $D_o^A p_o^A + D_o^B p_o^B$. By assumption, there are no transmission constraints and so both regions will have the same price.

Figure A1 about here

The shaded area to the right of the thick vertical axis shows how much of the available water is being used for hydro generation in this period. A second vertical axis is placed at the right-hand end of this area, and we measure production in region A from this point onwards. The upwards sloping line, $\phi_o g_o^{Th}$, gives the marginal cost of thermal output in the off-peak period. Measuring from the thick vertical axis, it gives the total supply in this period, taking the allocation of water as given. This intersects the total demand at the equilibrium price, p_o . The left-hand part of the diagram can be read as the mirror image of the first. This shows what happens in the peak period. The downwards-sloping lines give the demand curve for region B, $D_p^B p_p^B$, and for the two regions together, $D_p^A p_p^A + D_p^B p_p^B$, measuring leftwards from the thick vertical axis. The shaded area to the left of the vertical axis shows the amount of water allocated to generate hydro electricity in the peak period. The overall length of the shaded area represents the water constraint for the two periods together. Measuring from the left-hand end of the available water, the upwards-sloping line shows the marginal cost of

thermal generation in this period, $\phi_p g_p^{Th}$, and intersects with the overall demand curve at the equilibrium price, p_p .

With no constraints on how much water is carried over from one period to the other, or how much is used in either period (beyond the overall two-period constraint of the total water available), the equilibrium price is the same in both regions and in both periods. The vertical dashed lines show how much electricity is being used in each region – consumption in region B is from the vertical axis to the first dashed line (moving left in the peak period, right in the off-peak), while consumption in region A is given by the distance between the two dashed lines. The arrows below the horizontal axis show how much electricity is being produced by each type of plant in each period. There is much more hydro generation in the peak period than in the off-peak period, whereas the amount of thermal generation in each period is equal, given the assumption (in these figures) of identical marginal cost functions. The braces underneath the horizontal axis show that region B (with the hydro generation) is importing power in the off-peak period, and exporting it in the peak period, as predicted in section 3.

Figure A2 about here

Figure A2 shows what would happen if there was some wind generation in region A (our thermal country) in the off-peak period. This is given by the taller rectangle to the left of the (reduced) water allocated to hydro generation in this period. The marginal cost curve for thermal generation in this period starts at the right-hand end of the rectangle of wind output. This has moved to the right, relative to its position in figure A1, and so intersects with the (overall) demand curve at a lower equilibrium price than in that figure. Total consumption is therefore higher in the off-peak period than in the absence of wind generation.

The thermal marginal cost curve in the off-peak period moved to the right by less than the amount of thermal generation, because some water was reallocated from the off-peak period to the peak period. This has the effect of shifting the marginal cost curve for peak period thermal generation to the left, so that it, too, intersects with the peak demand curve (for the two regions combined) at the same, lower, equilibrium price as in the off-peak period. The amount of thermal generation is the same in each period, but less than in figure A1. Region B is now importing more power in the off-peak period (implying that Region A is indeed exporting some of its wind output), but it is also exporting more power in the peak period. The wind generation is partly absorbed by a reduction in thermal output, and part by an increase in consumption, in both regions and both periods. The higher consumption in region B means that its net exports of hydro power fall – the amount of water available is unchanged, and more of it is needed to cover domestic consumption. The change in region B's consumption is equal to the amount of wind-generated electricity that is permanently exported from region A, as opposed to being stored off-peak and returned at the peak.

It is straightforward to apply this framework to show the impact of wind generation in the peak period (which would reduce hydro exports at that time, and hence off-peak imports), or of lower thermal costs in the peak months (since electricity is co-produced with heat, which is in greater demand in winter), which could imply greater thermal production at the peak, and net imports to region B at that time.

Danish power and Nordic hydro

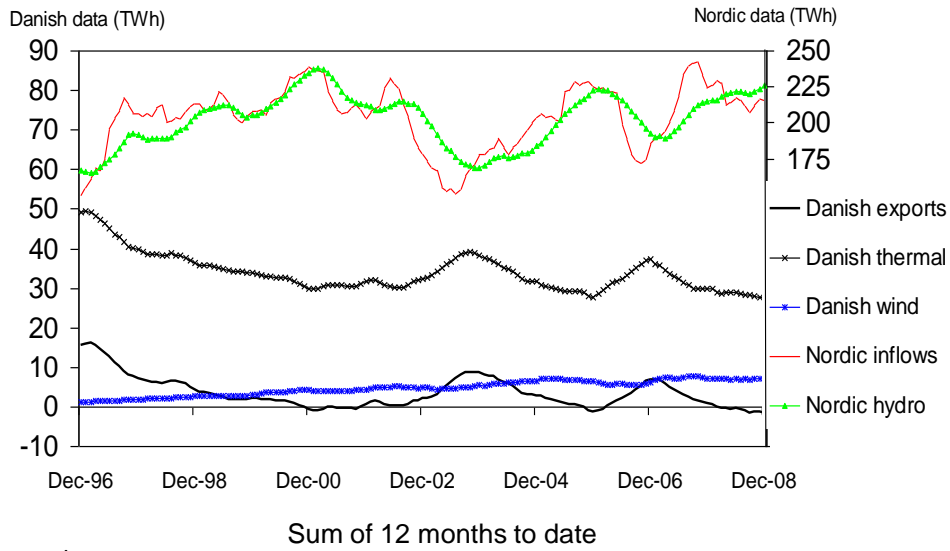


Figure 1

Monthly average output and consumption

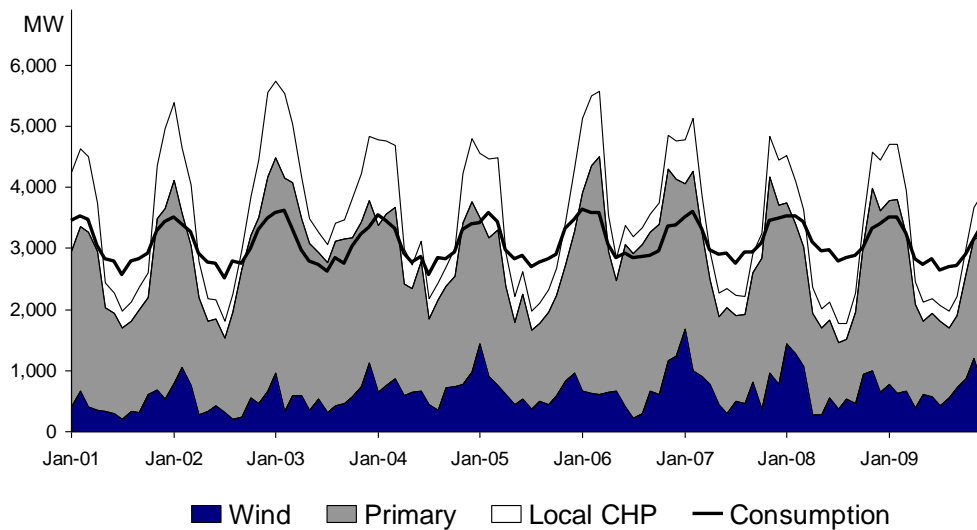


Figure 2

Daily pattern of output and consumption: winter

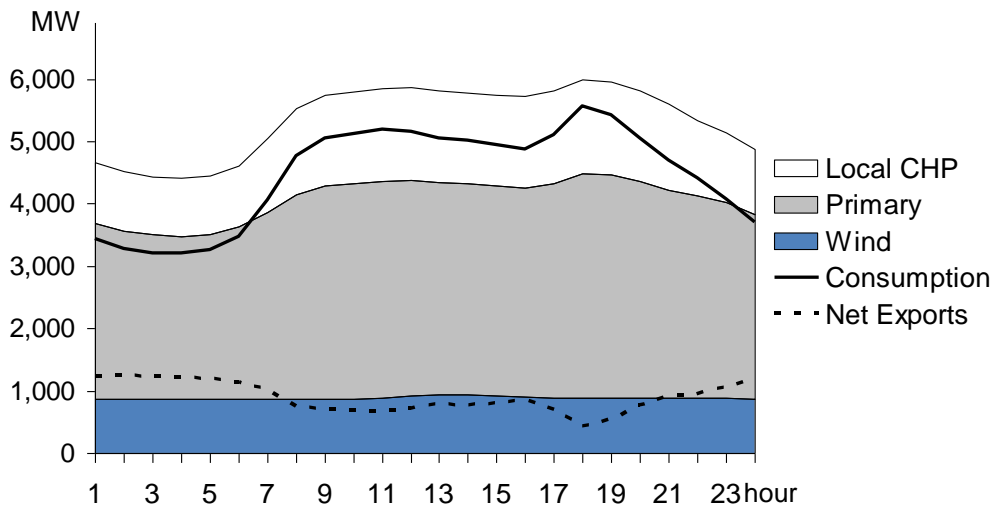


Figure 3a

Daily pattern of output and consumption: summer

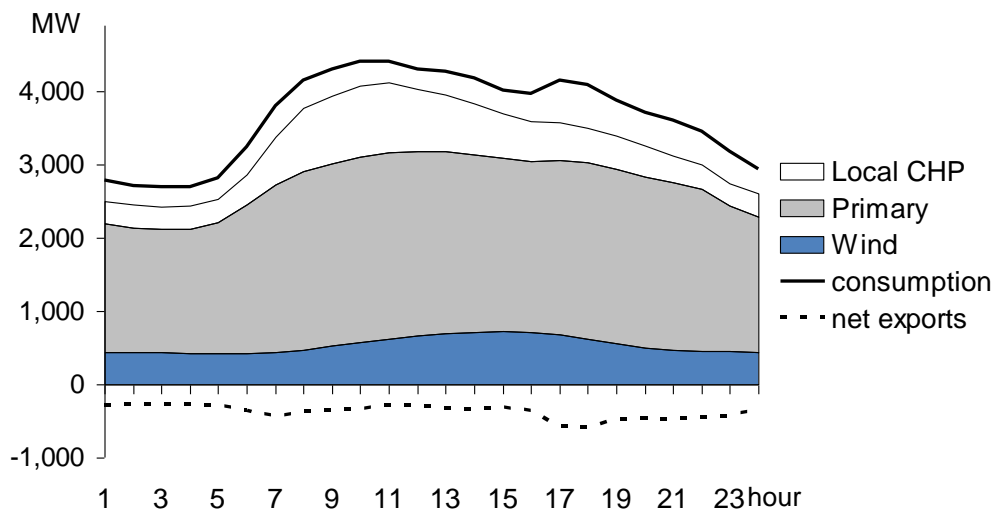
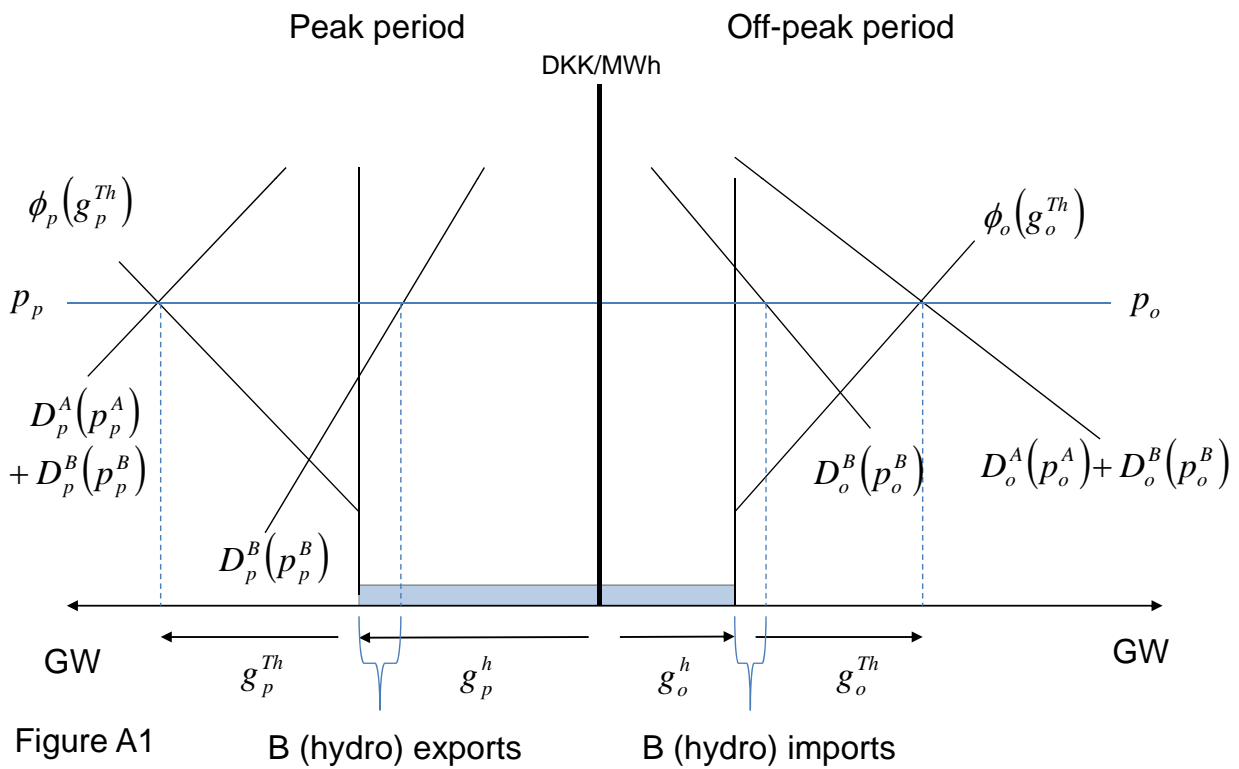


Figure 3b

A two-period hydro-thermal bathtub model



A two-period hydro-thermal bathtub model with wind

