

# Opportunity Cost Bidding by Wind Generators in Forward Markets: Closed Form Results

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**Abstract**—Wind generation must trade in forward electricity markets based on imperfect knowledge about its available capacity in real time, and the real time market prices. If a market has a single real time price, then a wind generator can forward-contract a central estimate for its output; however, where the real time prices differs for generators which are short and long, the optimal forward strategy will general be a bid curve based on the opportunity costs of charges and payments in real time. This paper presents closed-form results for wind’s optimal forward strategy. In the risk-neutral case, the optimal strategy is determined by the distribution for real-time wind availability, and the expected real time prices conditioned on the forward price and wind out-turn; this is simpler and more computationally efficient than formulations requiring specification of full joint distributions or a large set of scenarios. Informative closed-form examples may be derived for particular specifications of the wind-price dependence structure. The risk-averse problem is also examined. An analytical result is available for aversion to production volume risk; however, it is doubtful whether wind owners should be risk-averse with respect to the income from a single settlement period, given the large number of such periods in a year.

**Index Terms**—Power generation economics, Wind energy.

## I. INTRODUCTION

THE penetration of wind generation is increasing in power systems worldwide. Differently from conventional plant, whose availability assuming adequate fuel is mainly a matter of mechanical availability, the availability of wind capacity is primarily determined by the weather.

As a consequence, the statistical properties of wind capacity availability are very different from conventional plant, both on planning and operating timescales. On a planning timescale, this manifests itself as a probability distribution for available capacity at some distant time in the future. On an operating timescale, which will be considered here, the relevant probability distribution is that for wind out-turn conditional on the information available when decisions are taken; a wind generation owner cannot contract a level of output in a forward market and be sure that that precise amount will be delivered.

When generators contract to sell power in forward markets, imbalances between the forward-contracted volume and actual

output must be rectified in the real-time market. In some systems, there is a single real-time price seen by generators irrespective of whether they are short or long with respect to their forward-contracted volumes [add references]. In others (examples include the Great Britain [1], Scandinavian [2] and Iberian [3] markets), there are different real time prices for generators which are short or long. Forward market bidding by wind generation is thus a problem of optimisation under uncertainty regarding both the production out-turn and the real-time price, as the decision on trading policy must be made before precise information on these real-time quantities is available.

Optimal trading strategies for wind generation have received limited attention in the literature. [4] introduced the concept of optimising trading policy based on imbalance costs, in both risk-neutral and risk-averse formulations. [5] introduced a closed form expression for the optimal contract volume in the forward market given the forward and real time prices; the same optimisation approach was used in [6], which compares strategies based on point wind prediction and probabilistic wind forecasts, and also presents a brief discussion of risk-averse formulations. [2] then presented a stochastic mixed integer linear program (LP) formulation for this optimisation model, based on representing the uncertainty in wind out-turn using a finite number of discrete scenarios. This LP approach was extended from a two-market system (forward and real-time) to a three-market (day-ahead, adjustment and real-time) in [3]; in addition, a more efficient continuous LP formulation was presented, in which risk-averse behaviour could be included. Other relevant work includes a detailed discussion of utility metrics for risk-averse traders [7], assessment of the cost of wind forecast errors [8] and real time prices [9], advanced wind forecasting methods [10], operational strategies coordinating wind generation with reserves, storage and hydro (e.g. [11]–[13]), and designing power markets to minimise imbalance costs to wind generators [14].

This paper will present closed-form methods for optimising a wind generation owner’s trading strategy in a two-market (forward and real-time) system. These will require either evaluation of a single formula, or numerical solution of a specified integral or equation. Cases considered will include uncertain real time prices that are correlated with the forward price and wind out-turn, and also risk-averse bidding strategies. The benefits of closed form methods over previous approaches requiring solution of LP models are:

- *Transparency of results.* In closed form expressions and direct formulas it is much easier to identify what parameters drive the results obtained.
- *Computational efficiency.* The expressions presented here

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may be evaluated more quickly than LP models.

- *Efficiency of data specification.* The methods in this paper require only specification of conditional expectation values for the real time prices given the forward price and wind out-turn, as opposed to full joint distributions or a scenario tree.

This closed-form approach will be particularly valuable when the optimal wind-bidding strategy model must be embedded in a larger model, perhaps of the entire power market, or of generation investment decisions. In contrast, it is highly undesirable to have to embed a large stochastic linear program model for wind's strategy in such a whole-market model, and this paper provides computationally simpler alternatives.

Section II describes the simplest case of a single real time price. Section II then describes how multiple real time prices are used in some systems, and presents the optimal bid strategy in this case. Sections IV-VI illustrate how this may be applied, including cases with both independent and correlated wind output and real time prices, and also known or uncertain forward price. Finally Section VII generalises some of the results to the case of aversion to the risk of low real-time wind output. Throughout, examples are based on Great Britain data; the methods may also be applied to related market designs such as in Scandinavia [2].

## II. OPTIMAL VOLUME: SINGLE REAL TIME PRICE

### A. Forward Trading Based On Opportunity Cost

Like any other form of generation, a wind generator may contract in the forward market to deliver power in real time, whether this be through a bilateral trading or pool system. It might then deviate from its forward-contracted position, with any resulting system imbalance being rectified in the real time market. Differently from conventional generation, however, in real time wind will almost always generate at its maximum available capacity due to its very low marginal cost of production (exceptions may include curtailment on system security grounds at very high penetrations, or where there is limited transmission capacity). This paper assumes competitive markets in which individual wind owners are price-takers, and wind generates at maximum available capacity.

Wind's strategy in the *forward* market is a non-trivial problem based on uncertainty in real time wind out-turn and power prices. The *opportunity* cost of a forward contract influencing behaviour in the real time market is then relevant, rather than the negligible internal marginal cost of supplying capacity when available. Such opportunity cost-based trading is seen in a number of other aspects of power markets, e.g.

- Trading in ancillary services markets based on revenue forgone in the energy market [].
- Trading in one geographical area's market based on revenue foregone by not selling in other areas.
- Arbitrage between forward and real time markets.
- Hydro power with a finite water resource; bids would reflect sacrificed revenue at other times.
- Finite pollution permits for fossil fuel units
- Where a peaking unit is permitted a maximum number of starts per year under its maintenance contract.

### B. Single Real-Time Price

The simplest case of the forward contracting problem considers a wind operator selecting a contract volume  $q$  with forward price  $\pi_F$  (often called the system marginal price, SMP, or market clearing price, MCP), with a single real time (RT) price  $\Pi$  applied to all imbalances.  $\Pi$  and production out-turn  $W$  (conditioned on the available information at time of contracting) are random variables with joint probability density function  $f_{\Pi,W}(\pi, w)$ . The expected total income is then

$$\pi_F q + \int \int \pi(w - q) f_{\Pi,W}(\pi, w) d\pi dw. \quad (1)$$

Differentiating with respect to  $q$  to find the optimal bid  $\hat{q}$  (assuming no bounds on  $\hat{q}$ ):

$$\pi_F = \int \int \pi f_{\Pi,W}(\pi, w) d\pi dw = E[\Pi]. \quad (2)$$

This is a standard result that (assuming risk-neutral behaviour by market players) arbitrage between the RT and forward markets means that the forward price is the expected RT price.

Similar undramatic results are obtained from various generalisations of the single real time price problem, such as the a market where a number of identical wind owners bid simultaneously, and where the real time price is obtained endogenously within the model via a supply curve for conventional generation. These generalisations will not be discussed further in this paper, as while the final 'no arbitrage' result is not exciting, but the necessary algebra is quite complex.

## III. QUANTITY BIDDING: DIFFERENT SHORT AND LONG PRICES

### A. Introduction: GB Market Design

By contrast, in some power markets the prices felt by generators or trades in the real time market differ depending on whether they are short or long. Examples of this setup include British [1], Nordic [2] and Iberian [3] markets. If the wind generator is short/long in real time (i.e., delivers less/more than the forward-contracted volume), then this imbalance will be rectified at the short price  $\pi_S$  / long price  $\pi_L$ ; these may be respectively greater/less than the forward market price  $\pi_F$ . This is justified on the grounds that if a generator is in imbalance then the System Operator has to take actions to make good the imbalance of power, and that the cost of such actions should then be charged to those who caused them. It should be emphasised that in many, perhaps even a majority of, power markets worldwide wind is not penalised in this way for being short or long; the discussion in Section II-B of a single real time price then applies.

#### 1) Great Britain Market Design:

This section describes how prices are set in the Great Britain power market; the situation in Scandinavia is very similar (see Section II of [2].) The forward market in GB formally operates by bilateral trading (Chapter 10 of [15]); there is no centralised pool or power exchange. Trades between generators and demands are reported to the system operator (SO) by gate closure, which is one hour ahead of real time. There are however independent power exchanges, and the SO

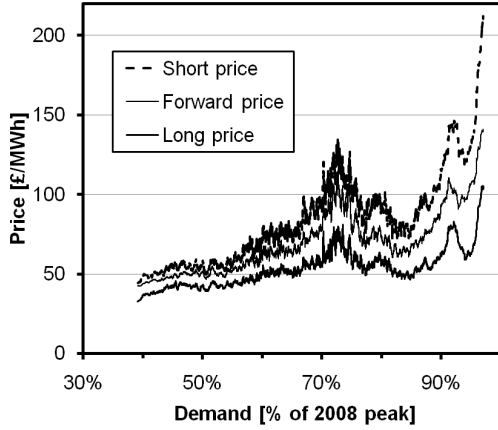


Fig. 1. Short, long, and forward Market Index Price (MIP) from the GB market in 2008. In order to reveal trends as demand varies, half hour periods are ranked in order of increasing demand, and a 101-point moving average is taken. The raw price data is taken from [17], and demand data from [18].

publishes a forward price index as a time series for each half-hour settlement period, based on data from these exchanges.

In real time, the market might either be long (total forward-contracted generation greater than out-turn demand) or short (contracted generation less than out-turn demand). Each period's short price (paid to trades which are short, called the system buy price in GB), and long price (paid to trades which are long, called in GB the system sell price) are defined as follows [16] (forward prices are based on market index data):

- Market short
  - Short price: average price of bids to increase output accepted by SO.
  - Long price: the forward price
- Market long
  - Short price: the forward price
  - Long price: average price of offers to reduce output accepted by SO (typically offers will be to *pay* the SO, due to the fuel costs saved.)

Imbalances in the opposite direction to the market are thus settled at the forward price; in designing the market, the justification for this was that such imbalances are helpful to the market, and should therefore not be penalised relative to the income from a perfectly balanced contract.

## 2) Great Britain Historic data:

The historic short, long, and forward market index prices from 2008 are displayed in Fig. 1. In order to reveal trends as demand varies, the half hour periods are ranked in order of increasing demand, and a 101-point moving average is then taken<sup>1</sup>. As each time series includes the data from all half hour periods (not differentiating between hours when the market was long or short), this graph presents the data as seen at the time of the forward market when players do not yet know whether the market will be long or short. As expected,

<sup>1</sup>Points are not plotted for the highest and lowest 50 demands, as the moving average would then be unbalanced. An alternative visualisation is then required, perhaps plotting the price averaged over all periods with demand above  $x$ , against demand level  $x$ .

the overall trend is for all the prices to increase as demand increases; however there are clear local maxima in the price-demand curves at around 72%, 79% and 92% of peak demand.

## B. Quantity Bidding: Risk-Neutral Case

### 1) Model Definition:

If the wind owner is willing to make decisions on the basis of its expected income, then it is termed risk-neutral; if it demands a risk premium due to the possibility of its income being less than the expected value then it would be termed risk-averse. We assume here that the wind generator is indeed risk neutral; due to the large number of time periods over which annual profit is measured any random fluctuations away from the mean will cancel to a good approximation, so this is equivalent to an implicit assumption is that there is no systematic error in the assumed wind out-turn distribution.

The forward price is assumed to be known precisely at the time the forward contract volume is chosen (bid curves for the more general case of uncertain forward price will be discussed in Section V.) The wind owner must therefore decide on its trading strategy in the forward market, given the uncertainty in its output in real time and the real time prices. The generation out-turn  $W$  and the real-time prices (short price  $\Pi_S$ , and long price  $\Pi_L$ ), conditional on the information available when trading in the forward market, are then modelled as random variables<sup>2</sup>. For application to the GB system these expected prices must take into account that if the wind owner is in imbalance in the opposite direction to the market then in real time it is charged the forward price. It is convenient to define the expected short and long penalties

$$E[\Delta_S|w, \pi_F] = E[\Pi_S|w, \pi_F] - \pi_F \quad (3)$$

$$E[\Delta_L|w, \pi_F] = \pi_F - E[\Pi_L|w, \pi_F], \quad (4)$$

where  $E[\Pi_{(S,L)}|w, \pi_F]$  are the expected real time prices conditioned on wind out-turn and forward price.

### 2) Derivation of Optimal Forward Volume:

The expected net revenue, which the risk-neutral wind owner seeks to maximise, is then

$$\begin{aligned} E[R] &= \pi_F q - \int_0^q E[\Pi_S|\pi_F, w](q-w)f(w)dw \\ &\quad + \int_q^{w^+} E[\Pi_L|\pi_F, w](w-q)f(w)dw \end{aligned} \quad (5)$$

$$\begin{aligned} &= \pi_F E[W|\pi_F] - \int_0^q E[\Delta_S|\pi_F, w](q-w)f(w)dw \\ &\quad - \int_q^{w^+} E[\Delta_S|\pi_L, w](w-q)f(w)dw, \end{aligned} \quad (6)$$

where  $f(w)$  is the estimated probability density function for real time output. Differentiating, the optimal volume  $\hat{q}$  is

$$\int_0^{\hat{q}} E[\Delta_S|\pi_F, w]f(w)dw = \int_{\hat{q}}^{w^+} E[\Delta_S|\pi_F, w]f(w)dw. \quad (7)$$

<sup>2</sup>Throughout, the convention that lower case letters (e.g.  $w$ ) denote numbers and capital letters (e.g.  $W$ ) denote random variables; the probability density function for  $W$  is  $f_W(w)$ .

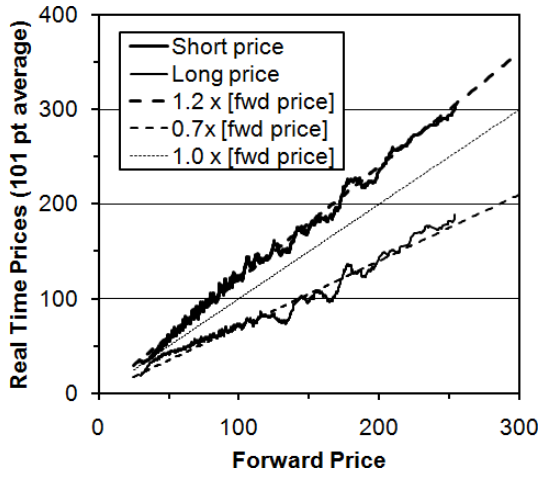


Fig. 2. Expected real time prices as a function of forward price in Great Britain. These expected RT prices conditioned on the forward price are calculated by ordering half hour periods by forward price and taking a 101 point moving average.

The key features of this expression are as follows:

- In order to evaluate the optimal forward contract volume, it is sufficient to specify just the expected real time prices/penalties, conditional on the forward price and wind out-turn. This is much more straightforward than specifying a full joint distribution for prices and wind out-turn, or specifying a large set of discrete scenarios.
- (7) may be solved using direct numerical methods (numerical integration and equation solving). It is not necessary to use a mathematical optimisation algorithm, which is required in an LP formulation such as [2], [3].

#### IV. APPLICATION I: UNCORRELATED WIND OUT-TURN AND REAL-TIME PRICES

##### A. Expression for Optimal Forward Contract Volume

If the short and long penalties are independent of the wind out-turn, then the expression for the optimal forward contract quantity  $\hat{q}(\mu_F)$  at price  $\pi_F$  simplifies to

$$p(W \leq \hat{q}(\pi_F)) = \frac{E[\Delta_L|\pi_F]}{E[\Delta_S|\pi_F] + E[\Delta_L|\pi_F]}, \quad (8)$$

where  $E[\Delta_{(S,L)}|\pi_F]$  is the expected value of  $\Delta_{(S,L)}$  given forward price  $\pi_F$ .

This expression is similar to those in [5], [6]; the derivation presented here makes it explicit that expected penalties given the forward price should be used, clarifying the statement in [6] that real time price ‘forecasts or estimates’ must be used.

The formula for the optimal volume is completely closed-form, with the benefits of transparency and minimal computational effort which this brings.

##### B. Examples

###### 1) Wind Owner in Great Britain:

Expected real time prices conditioned on the forward price for Great Britain in 2008 are displayed in Fig. 2; the data used is the same as in Fig. 1. These expected RT prices

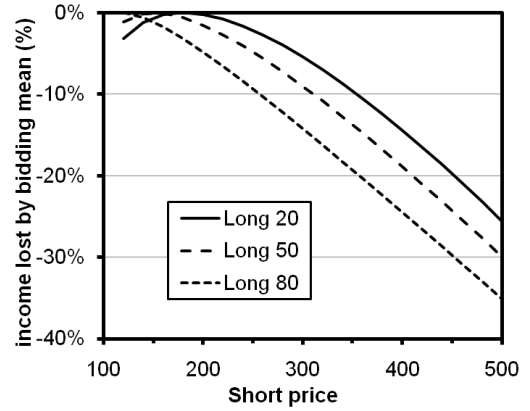


Fig. 3. Percentage decrease in expected revenue from forward-contracting expected out-turn, compared to revenue from the optimal strategy presented here. The out-turn wind load factor conditioned on the information available at time of forward contracting follows the  $N(0.5, 0.125)$  distribution.

are calculated by ordering half hour periods by forward price and taking a 101-point moving average. This carries an implicit assumption that the expected RT prices depend only on the forward price, with no seasonal and diurnal effects; comparison with plots considering one season only shows that this is reasonable for an illustrative example.

Over a wide range of forward prices the expected real time short and long prices are approximated well by  $1.2\pi_F$  and  $0.7\pi_F$  respectively, giving  $E[\Delta_S|\pi_F] \simeq 0.2\pi_F$  and  $E[\Delta_L|\pi_F] \simeq 0.3\pi_F$ . Fig. 2 shows also the line RT price = forward price (i.e. no penalties); the vertical distance between that line and the lines  $1.2\pi_F$  and  $0.7\pi_F$  is equal to the penalty for being short/long. Over this range, the ratio of the expected short and long penalties remains constant, and the optimal contract volume is given by  $p(W \leq \hat{q}) = 0.6$ . The optimal contract quantity being greater than the median out-turn is consistent with the expected long penalty exceeding the expected short penalty; if the penalties were symmetric then the optimal contract quantity would be the median out-turn. The actual MW optimal volume depends on the probability distribution for wind out-turn.

###### 2) Benefits of Optimal Strategy:

The benefits of following this optimal strategy, as opposed to forward-contracting expected out-turn, are demonstrated in Fig. 3. The wind load factor here is Normally distributed with mean 0.5 and standard deviation 0.125. This distribution is consistent with the day-ahead forecast accuracy used in [2], is similar to four-hour ahead data from Great Britain (the forecast SD in GB is however about half the size at gate closure 1 hour ahead).

The forward price is assumed to be 100 units; expected long prices down to a factor of 5 below this, and expected short prices up to a factor of 5 above, are considered. It may be seen that for small expected penalties (differences between forward and expected real time prices), the benefits of the optimal strategy are not very great. With a moderate short price, a very low long price alone does not provide benefits of more than a few percent over contracting expected out-turn

volume. However, if the expected short price is very high, then the benefits of the optimal strategy can be very substantial indeed. Variation in the short price has the greater potential effect, as there is no ceiling on it; the long price is limited below by zero, so the size of the long penalty and its effect on the benefits of optimal bidding strategies is limited

## V. FORWARD MARKET BID CURVES

### A. Uncertain Forward Price in a Pool or Exchange

If the forward market price is known with certainty at the time the bidding strategy is devised, then the above approach of contracting a fixed volume is sufficient. Also, as seen in (8), if the ratio of expected short and long penalties is independent of forward price, then the same forward contract volume will be optimal independently of forward price.

In general, however, the optimal forward contract volume can depend on the out-turn forward price (and the resulting expected penalties). If a bid must be submitted to a pool or power exchange, then at the time the bid is devised the forward price cannot be known with certainty; account should be taken of this when devising the form of the bid.

### B. Properties of the Dependence of Optimal Quantity on Forward Price

The optimal bid quantity equation may be rearranged as

$$p(W \leq \hat{q}(\pi_F)) = \left(1 + \frac{E[\Delta_S|\pi_F]}{E[\Delta_L|\pi_F]}\right)^{-1}. \quad (9)$$

For a given forward price, the optimal quantity is thus determined by the ratio of the expected short and long penalties. If, as the forward price increases the ratio  $E[\Delta_S|\pi_F]/E[\Delta_L|\pi_F]$

- increases, then the optimal forward volume decreases
- is fixed, then the optimal forward volume stays constant
- decreases, then the optimal forward volume increases

Fig. 4 shows examples for each of these, where  $E[\Delta_S|\pi_F] = 0.2\pi_F$ , and  $E[\Delta_L|\pi_F] = 0.3\pi_F + x$  where  $x = 0$  (ratio constant), 10 (ratio increasing) and -10 (ratio decreasing).

### C. Derivation of Bid Curves

A bid curve for a pool or exchange may be derived by swapping the axes on a plot of optimal quantity versus forward price; the bid curve indicates what price is required for the generator to supply a given quantity. For the cases where the ratio of expected short to long penalties is constant with price (implying a fixed optimal volume independent of forward price,  $x = 0$  in Fig. 4), or decreasing (implying optimal volume increasing with forward price,  $x = -10$ ), the bid curve will have a typical non-decreasing form.

The case where the ratio of expected short to long penalty increases as the forward price increases, however, implies a bid curve where price required decreases as quantity increases ( $x = 10$  in Fig. 4). Where the bid strategy reflects internal short run marginal costs of supply, this could not occur, as then

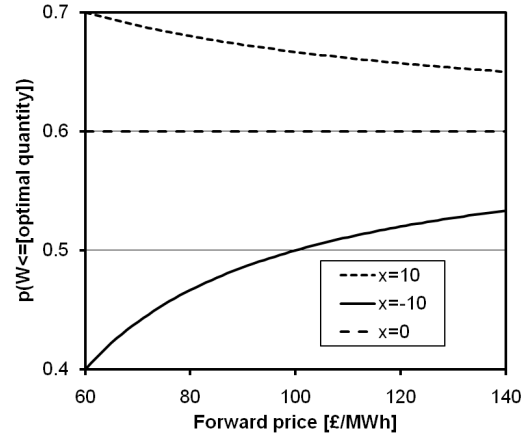


Fig. 4. Optimal bid volume as a function of forward price for expected short price  $E[\Delta_S|\pi_F] = 0.2\pi_F$  and expected long price  $E[\Delta_L|\pi_F] = 0.3\pi_F + x$ , where  $x = (-10, 0, 10)$  /MWh.

the generator would never wish to decrease its supply quantity as the price increases. Indeed, in some markets only non-decreasing bid curves are permitted, as a protection against exercise of market power. For the situation here where the forward market strategy is not based on internal supply costs, as demonstrated in (8) and Fig. 4, the bid curve can in principle have negative gradient for certain penalty structures. It should be further noted that the forward contract volume does not usually affect wind's behaviour in real time, when it will generate at the maximum available output level.

## VI. APPLICATION II: CORRELATED WIND OUT-TURN AND REAL TIME PRICES

This subsection presents two examples of how the method of Section III-B may be used when the wind out-turn and real time prices are not independent. Both provide closed-form results for the optimal bid volume, which provide insight into drivers of the optimal bidding behaviour.

### A. Expected Penalties Based on Thermal Supply Function

Suppose that the GB wind owner's portfolio is well-distributed across the country, so that if its output is  $w$  then for some constant  $N$  the total GB wind output is  $Nw$ . If there were a single real time price in a perfectly competitive market, this would then be  $\pi_R = \pi_T(d - Nw)$ , where  $\pi_T(q)$  is the marginal cost of supply by thermal generation at quantity  $q$ .

As described above, the GB market does not have a single real time price. However, one can gain insight into how wind out-turn affects the optimal bid strategy by assuming a dependence of the short and long penalties on the wind out-turn based on the competitive case.

The thermal supply quantity resulting from the expected wind out-turn is  $q = d - N\mu_W$ . Making a quadratic approximation to the thermal supply function about this,

$$\pi_T(q) \simeq \pi_T(d - N\mu_W) + c_1(q - (d - N\mu_W)) + c_2(q - (d - N\mu_W))^2. \quad (10)$$

For a convex thermal supply function, the constants  $c_1$  and  $c_2$  are positive. Assuming the same form for the dependence of the short and long prices on the wind out-turn,

$$E[\Delta_S|\pi_F, w] = E[\Delta_S|\pi_F, w = \mu_W] - c_1 N(w - \mu_W) + c_2 N^2(w - \mu_W)^2 \quad (11)$$

$$E[\Delta_L|\pi_F, w] = E[\Delta_L|\pi_F, w = \mu_W] + c_1 N(w - \mu_W) - c_2 N^2(w - \mu_W)^2. \quad (12)$$

For simplicity, this assumes that if the wind generator is well below its expected real time output then the market is definitely short, and similarly for high output; for the illustrative example here this assumption is reasonable at high wind penetrations.

The optimal bid quantity  $\hat{q}$  is then given by

$$p(W \leq \hat{q}) = \frac{E[\Delta_S|\pi_F, W = \mu_W] - c_2 \sigma_W^2}{E[\Delta_S|\pi_F, W = \mu_W] + E[\Delta_L|\pi_F, W = \mu_W]}. \quad (13)$$

Comparison with the expression for uncorrelated real time prices and wind-out-turn (8) shows that the  $-c_2 \sigma_W^2$  term acts as a correction to the numerator of the expression for  $p(W \leq \hat{q})$ , decreasing the optimal forward contract volume if the thermal supply curve is convex (i.e. positive second derivative). This fits with intuition, as with a convex thermal supply curve the RT prices increase more when wind is short than they decrease when wind is long, hence wind should tend more towards being long to avoid this enhanced short penalty. This substantial correction term, of a kind more normally seen in results assuming a high degree of market power, appears despite the assumption that the wind owner's behaviour does not affect the forward market price.

In competitive markets, it is unusual to see an assumption that one player knows in advance that all competitors will make the same decision, and know what that decision will be. However, in this case the wind owners do not really make a decision regarding their output; rather they all inevitably generate at the maximum available capacity due to having negligible short run marginal cost of supply in real time. The correction  $-c_2 \sigma_W^2$  above follows from an assumption that the wind owner's out-turn load factor has the same probability distribution as the overall GB load factor; in fully realistic cases where the degree of correlation is less, then this correction term will be reduced somewhat.

### B. Dependence Structure From a Multivariate Normal Distribution

This example provides a more flexible means of deriving the dependence structure, while still providing a direct method of obtaining the optimal forward contract quantity through numerical solution of an equation.

The wind out-turn is assumed to follow a Normal distribution, with probability density function

$$f_W(w) = \frac{1}{\sigma_W \sqrt{2\pi}} \exp\left[-\frac{(w - \mu_W)^2}{2\sigma_W^2}\right]. \quad (14)$$

The expected short penalty given wind out-turn  $w$  is then

$$E[\Delta_S|W = w] = \mu_S + \left(\frac{\rho_S \sigma_S}{\sigma_W}\right) (w - \mu^W). \quad (15)$$

A similar expression holds for the long penalty in terms of parameters  $\rho_L$ ,  $\mu_L$  and  $\sigma_L$ . In a multivariate Normal distribution, these would be the correlation coefficient between wind-out-turn and prices, and the mean and standard deviation of the prices, but this assumption of the distribution's precise form is not required for the results which follow. The specification of parameters in (15) must account for the possibility of the real-time price being set equal to the forward price if the wind owner is in imbalance in the opposite direction to the market.

Substituting in (7), the following equation for the optimal forward contract volume  $\hat{q}$  is obtained, assuming that the probability wind outputs near minimum or maximum is negligible:

$$(\mu_S + \mu_L)p(W \leq \hat{q}) = \mu_L + \frac{(\rho_L \sigma_L + \rho_S \sigma_S)}{\sqrt{2\pi}} \exp\left[-\frac{(\hat{q} - \mu_W)^2}{2\sigma_W^2}\right] \quad (16)$$

Once again, this may be regarded as adding correction terms to the 'uncorrelated' case (8). It does not provide a completely closed-form expression for the optimal quantity, but this may be evaluated by direct numerical solution of the equation. It is possible to analyse the effects of a relationship between the wind out-turn and prices as follows:

- If the short penalty is negatively correlated with the wind out-turn (which is likely to be the case), then compared to the independent case the wind owner reduces the forward contract volume at a given price. This behaviour mitigates the consequences of an increased short price when the wind out-turn is low.
- If the long penalty is positively correlated with the wind out-turn (which is likely to be the case), then compared to the independent case the wind owner increases the forward contract volume at a given price. This behaviour mitigates the consequences of a decreased long price when the wind out-turn is high.

As a result, the behaviour of wind relative to the independent case is determined by the relative volatilities of the short and long prices, and the degree to which they are correlated with the wind out-turn.

## VII. RISK-AVERSE TRADING BEHAVIOUR

### A. VaR and CVaR

The previous results have all been for risk-neutral behaviour, i.e., where the wind owner seeks to maximise expected revenue, without any consideration of how far below that expected value the revenue can decrease. The most common measures used to analyse risk-averse trading strategies are Value at Risk (VaR) and Conditional Value at Risk (CVaR) [19].

The VaR for the revenue  $R$  at the  $\alpha$  confidence level ( $\alpha$ -VaR) is defined as the revenue  $r_\alpha$  such that  $p(R \leq r_\alpha) = 1 - \alpha$ . The CVaR for the revenue  $R$  at the  $\alpha$  confidence level ( $\alpha$ -CVaR) is then defined as the expected revenue conditioned on the revenue being below  $\alpha$ -VaR, i.e. in mathematical notation  $\alpha$ -CVaR is  $E[R|R \leq r_\alpha]$ .

CVaR is more commonly used as the objective function when optimising risk-averse strategies, because unlike VaR it exhibits appropriate mathematical behaviour under general conditions ('coherence') [19], and may be included within LP

formulations [20]. CVaR also allows a smooth transition from risk-neutral behaviour ( $\alpha = 0$ ) to increasing degrees of risk aversion, by varying the parameter  $\alpha$ .

### B. Risk-averse Forward Trading Strategy

Our approach of deriving closed-form expressions can be extended to risk-averse behaviour, provided that the long and short real time prices can be treated using expected values. The wind owner is then assumed averse to quantity risk but not price risk. Aversion to price risk requires evaluation of various double integrals of the joint probability distribution for prices and wind out-turn. While this full risk-averse problem is numerically tractable, the degree of transparency seen in other results from this paper is not available.

Given the large number of time periods over which a trading strategy will be used, in practice the degree of risk aversion which wind owners demonstrate in any given period may be limited (in the long run, a large degree of cancellation is to be expected between the consequences of below-average and above-average wind out-turns for single periods.) A case might be made that aversion to price-risk is equally important as quantity risk, due to the possibility of extreme spikes in the short price. However, the greatest long-run risk could well be systematic error in the estimated wind distribution, so that the calculated optimal quantity has a bias towards being either above or below the true optimum; in this case quantity risk aversion would be more important.

Based on this limited picture of aversion to wind out-turn risk, assuming known forward price  $\pi_F$  and real time prices independent of wind out-turn,

$$\begin{aligned} \text{CVaR}_\alpha &= \pi_F q - \frac{E[\Pi_S|\pi_F]}{1-\alpha} \int_0^q (q-w)f(w)dw \\ &\quad + \frac{E[\Pi_L|\pi_F]}{1-\alpha} \int_q^{w_\alpha} (w-q)f(w)dw, \end{aligned} \quad (17)$$

where  $p(W \leq w_\alpha) = 1 - \alpha$ , and it is assumed that the optimal forward quantity is less than  $w_\alpha$  (which is reasonable for small  $\alpha$ ). Differentiating to obtain the optimal bid volume  $\hat{q}$ :

$$p(W \leq \hat{q}) = (1 - \alpha) \frac{E[\Delta_L|\pi_F]}{E[\Delta_S|\pi_F] + E[\Delta_L|\pi_F]}. \quad (18)$$

The optimal forward contracted volume decreases as the degree of risk aversion increases; this is as expected, as risk-averse behaviour assigns less utility to payments for being long in real time, than it does to penalties for being short.

### C. Example

This section presents an illustrative example, with parameters based on wind and price data from Great Britain. The short and long prices are assumed to be respectively 120% and 70% of the forward price, based on the data in Fig. 2. The out-turn wind load factor is assumed to have a Normal distribution with mean 0.42 and standard deviation 0.12; these parameters are based on the variation in wind out-turn for hours in 2008 when the 4 hour ahead load factor was between 40% and 50%.

Results using the wind-volume-risk-averse strategy (18) are shown in Fig. 5. With the risk aversion parameter below 50%,

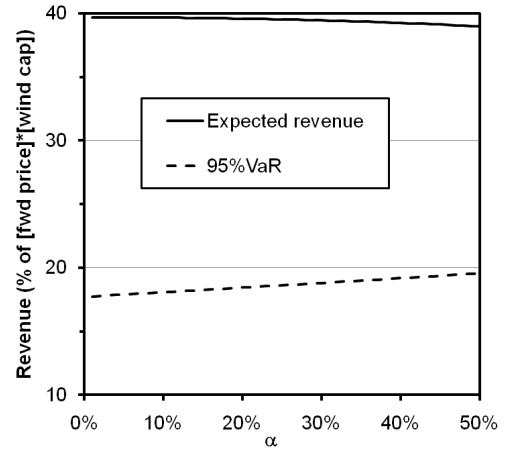


Fig. 5. Expected revenue, and 95%-VaR, for varying risk aversion parameter  $\alpha$ . The revenue is measured in units of [forward price] multiplied by [installed wind capacity].

a small decrease (less than 2%) in the expected revenue is seen. As a measure of the positive effect of the risk-averse strategy, 95%-VaR has also been plotted (VaR is used instead of CVaR here because of its more intuitive interpretation) Over the same range of  $\alpha$ , the gain in 95% VaR is slightly larger in absolute terms (and considerably larger in relative terms) than the decrease in expected revenue; a similar result has been noted in [3]. However, as discussed in Section VII-B, it is doubtful whether looking at individual periods in isolation is the right way to consider risk aversion for this problem.

## VIII. CONCLUSIONS

This paper has presented new closed-form expressions for determining wind generation's optimal strategy in forward markets, including cases where the real time prices and the wind owner's real-time production are not independent, and where the wind owner is averse to production volume risk. Examples based on data from Great Britain have been presented, and the methods are directly applicable to other markets such as those described in [2], [3].

The methods presented here require specification only of the expected real time prices given the forward price and wind out-turn; this is much more straightforward than specifying a full joint distribution or a representative finite set of scenarios. The new closed-form approach is also much more computationally efficient, and transparent in interpretation, than previous linear programming formulations; Moreover, in the risk-neutral case the new approach does not sacrifice any generality in the optimisation problem for a single period.

The benefits of risk-averse behaviour have also been discussed, concluding that given the large number of (often sub-hourly) settlement periods in a year, it is doubtful whether wind generation should optimise its behaviour based on aversion to revenue risk for any single period. The risk-neutral results presented here would then apply; if a full analysis of risk aversion is required, then the consequences of systematic error in the probabilistic wind forecasts should be studied.

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